

Fermion Masses from SO(10) Hermitian Matrices

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Abstract

Masses of fermions in the SO(10) 16-plet are constructed using only the 10, 120 and $\overline{126}$ scalar multiplets. The mass matrices are restricted to be hermitian and the theory is constructed to have certain assumed quark masses, charged lepton masses and CKM matrix in accord with data. The remaining free parameters are found by fitting to light neutrino masses and MSN matrices result as predictions.

1 Introduction

The simplest SO(10) treatment of fermion masses, with the fermions being in SO(10) 16-plets, uses the composition of 256 pairings of fermions into the 10, 120 and $\overline{126}$ SO(10) representations. There are 3 different 16-plets for the 3 generations of fermions and the fermion pairs are coupled to scalar (generalized Higgs) bosons in the same SO(10) representations with 3×3 Yukawa coupling matrices; and when the scalar bosons develop vevs the 3×3 fermion mass matrices are generated. The philosophy in this paper is to assume fermion masses only arise through coupling into these representations.

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In the coupling to the $\{10\}$ and $\{\overline{126}\}$ bosons $SO(10)$ gives equal coupling to the mass terms $\bar{\psi}_A\psi_B$ and $\bar{\psi}_B\psi_A$ where A, B label different 16-plets of fermions. Here they are taken to be generation labels and thus the Yukawa coupling matrices can be taken to be symmetric without loss of generality; contrairiwise for the $\{120\}$ the coupling of the mass terms $\bar{\psi}_A\psi_B$ and $\bar{\psi}_B\psi_A$ are of equal magnitude but opposite signs so that the $\{120\}$ Yukawas can be taken anti-symmetric [1]. For this, chiral, theory there is no constraint, in principle, that the Yukawa or mass matrix elements be real.

In the very many $SO(10)$ fermion mass investigations, ranging from simple hypotheses to complicated varieties of SUSY GUTS, there has been until recently a preponderance of hypotheses with no $\{120\}$ Higgs particles [2, 3, 4, 5, 6]. This rather arbitrary neglect seems to have been motivated, not unnaturally, by the search for simplicity, solvability and predictive power. Latterly, partly influenced by neutrino data, there have been more papers taking account of the $\{120\}$ [7, 8, 9] - some of these still giving a predominant role to $\{10\}$ and $\{126\}$ *ab initio*. This present paper is an example of the $\{120\}$ filling a necessary role, that of supplying the CP non-invariant, imaginary part, of the CKM matrix.

There have been continual developments of $SO(10)$ type theories - among other reasons there have been the demands of data matching. Beginning with theories having real Yukawa coefficients and real vacuum expectation values these have ranged to much more sophisticated models such as many having vevs with arbitrary phases to be determined from data extrapolated to a GUT scale (as for example in refs. [2, 8]). In this paper a simplification from general possibilities is chosen - that is the contribution to the fermion mass matrices from the $\{10\}$ and $\{\overline{126}\}$ to be real and symmetric while that from the $\{120\}$ be pure imaginary and anti-symmetric, resulting in Hermitian mass matrices. That condition on the $\{10\}$ and $\{\overline{126}\}$ is familiar, especially in early papers, and can be presented as 'naturally' implemented using real Yukawas; in the formalism given in the next section the above condition on the $\{120\}$ contribution can be presented as equally 'natural'; that is the mathematical formalism, as outlined in section 3, suggests the hermiticity as the simplest choice.

There is also an important physical motivation for the choice of hermiticity, this being the association in the formalism with parity invariance at high energy. If the γ_5 terms are absent in the mass terms these are straightforwardly parity invariant, giving a limiting case of the chiral theory for the quark and charged lepton equations. And the absence of the γ_5 terms gives

the hermiticity and then also the formalism yields the mass contributions of the $\{10\}$ and $\{126\}$ as CP conserving and that of the $\{120\}$ as CP violating. It can thus be said that there exists at least one basis (implicit in the formalism) in which the above mass matrices are hermitian. The one exception to high energy parity invariance and hermiticity in the mass terms of the model results indirectly, that is in the light neutrino masses from the contribution of the original seesaw (often known as 'Type I') in which the product of three hermitian mass matrices yields a non-hermitian matrix. The details for the model are in section IV below and the Appendix. (As has been emphasized recently [12] parity restoration in proceeding to high energies was one of the main motivations for the left-right symmetry principle which features in the next section.)

The hermiticity, as derived below, is more precisely that of 3×3 flavour matrices associated with the symmetry representations, linear superpositions of which form the hermitian mass matrices of the quarks and charged leptons - and one other contribution to the light neutrino mass matrix

At least two papers [10, 11] incorporating the 120 along with hermitian mass matrices have previously been published. Both of these are set in MSSM theory and one in particular [10] has much detailed discussion of the various symmetry breakings in the MSSM context. The present paper is based on SO(10) but has no commitment to supersymmetry or details of a Higgs mechanism or other higher theory.

The experimental data input to this model are the quark and charged lepton masses and the CKM matrix; so the parameters of the model have to be chosen to accomodate these numbers some of which carry considerable uncertainty; there can be further uncertainties from extrapolation to higher energies. The vital question then is can one adjust the very few remaining *completely* unknown parameters so that the model be compatible with the likely masses and MSN matrix of the three light neutrinos. In this paper the SO(10) vevs (or substitute mechanism) giving rise to the masses are parameters to be chosen to match data and no particular Higgs potential (or other mechanism) is postulated..

In section II the SO(10) Clifford algebra formalism for the fermion masses is given. Section III outlines the calculation with the Hermitian matrix hypothesis and gives the resulting mass formulae in terms of Yukawa coefficients and scalar vacuum expectation values; while in section IV the assumptions of the theory allow the expression of the mass matrices in terms of Hermitian matrices and real ratios of vevs and also allow the incorporation of the quark

and charged lepton mass data and CKM complex matrix data. Section V deals with the neutrino masses with emphasis on simple examples. One illustrates tri-bimaximal mixing arising from neutrino masses suggested by the experimental data.

2 SO(10) and its 16-plets

The Clifford algebra formalism of SO(10) [1] is based on ten gamma matrices $(\Gamma_1, \dots, \Gamma_{10})$ giving the 45 generators $\Sigma_{\mu\nu} \equiv [\Gamma_\mu, \Gamma_\nu]/2i$. The gammas can also be expressed through the creation and annihilation operators $\chi_j, \chi_j^\dagger, (j = 1, \dots, 5)$ where

$$\Gamma_{2j-1} = -i(\chi_j - \chi_j^\dagger), \Gamma_{2j} = (\chi_j + \chi_j^\dagger)$$

The fermion 16-plet, ψ_+ of positive 10d chirality is expressed as

$$|\psi_+\rangle \equiv |0\rangle \psi_0 + \frac{1}{2} \chi_j^\dagger \chi_k^\dagger |0\rangle \psi_{jk} + \frac{1}{24} \epsilon^{jklmn} \chi_k^\dagger \chi_l^\dagger \chi_m^\dagger \chi_n^\dagger |0\rangle \tilde{\psi}_j \quad (1)$$

where $\psi_0, \psi_{jk}, \tilde{\psi}_j$ are 2-component left-handed spinors of the particular generation. The assignment to the leptons and the quark $SU(3)_{\text{colour}}$ triplets is, in an obvious notation with a, b being indices 1, 2, 3 and d, u being colour triplets: $\tilde{\psi}_a \approx (d_R)^c; \psi_{a5} \approx d_L; \psi_{ab} \approx (u_R)^c; \psi_{a4} \approx u_L; \tilde{\psi}_4 \approx e_L^-; \psi_{45} \approx (e_R^-)^c; \tilde{\psi}_5 \approx n_L; \psi_0 \approx -(n_R)^c$ where n_R denotes right-handed (heavy) neutrinos.

The conjugate of equation (1) transforming appropriately under SO(10) is

$$\langle \psi_+^\star | B_\Gamma \equiv \langle \tilde{0} | \psi_0^T + \psi_{ij}^T \langle \tilde{0} | \left(\frac{1}{2} \chi_j^\dagger \chi_i^\dagger \right) + \tilde{\psi}_j^T \langle 0 | \chi_j \quad (2)$$

where $B_\Gamma \equiv i\Gamma_1\Gamma_3\Gamma_5\Gamma_7\Gamma_9$ and $\langle \tilde{0} | \equiv \langle 0 | \chi_5\chi_4\chi_3\chi_2\chi_1$.

Let the suffices A and B be generation indices for the 3 generations. Then, letting C be the charge conjugation matrix

$$\langle A | X | B \rangle \equiv \langle \psi_{+A}^\star | B_\Gamma C^{-1} X | \psi_{+B} \rangle \quad (3)$$

with $X = \Gamma_\mu$ or $X = \Gamma_\mu\Gamma_\nu\Gamma_\rho$ or $X = \Gamma_\mu\Gamma_\nu\Gamma_\rho\Gamma_\sigma\Gamma_\tau$ form the SO(10) representations 10 or 120 or 126 respectively. Under the action of a symmetry group generator $\Sigma_{\mu\nu}$

$$\langle A | X | B \rangle \rightarrow \langle A | [X, \Sigma_{\mu\nu}] | B \rangle. \quad (4)$$

Multiplying respectively by scalar fields ϕ_μ , $\phi_{\mu\nu\rho}$, $\phi_{\mu\nu\rho\sigma\tau}$ of the same representations gives SO(10) invariants, for example

$$\langle A | \Gamma_\mu | B \rangle \phi_\mu \equiv \langle \psi_{+A}^* | B_\Gamma C^{-1} \Gamma_\mu | \psi_{+B} \rangle \phi_\mu$$

which for colourless neutral vevs contribute to the elements of the 3 by 3 generation mass matrices of the quarks and leptons [1].

It is convenient to classify these vevs as the neutral colourless members of multiplets of the Pati-Salam subgroup of SO(10). Selecting the generators formed by $(\Gamma_1, \dots, \Gamma_6)$ gives the 15 generators of an SO(6) subgroup, and likewise $(\Gamma_7, \dots, \Gamma_{10})$ give the 6 generators of an SO(4) subgroup. These realise the $SO(6) \times SO(4)$, otherwise $SU(4)_c \times SU(2)_L \times SU(2)_R$, Pati-Salam subgroup of SO(10), the 3 + 3 generators of $SU(2)_L \times SU(2)_R$ being linear combinations of the 6 generators of SO(4). In terms of the creation and annihilation operators ($j = 1, 2, 3$) can give SO(6) and ($j = 4, 5$) can give SO(4). We are concerned with transition operators X, a subset of those above, which form $SO(6) \times SO(4)$ (equivalently $SU(4)_c \times SU(2)_L \times SU(2)_R$) invariant elements

$$\sum_X \langle A | X | B \rangle \phi_x$$

by coupling to the scalar field ϕ_x which transforms in the same $SU(4)_c \times SU(2)_L \times SU(2)_R$ representation as $\langle A | X | B \rangle$. The neutral, colour singlet, subset X_0 of X, coupling to vevs ϕ_{x_0} and multiplied by Yukawa coupling constants $Y_{AB}(X_0)$

$$\sum_{X_0} \langle A | X_0 | B \rangle \phi_{x_0} Y_{AB}^\rho(X_0) \quad (5)$$

yields contributions to 3 by 3 mass matrices. Here ρ , being 10 or 120 or 126, denotes the SO(10) representation to which X_0 belongs; thus the Yukawas are those appropriate to an unbroken SO(10) symmetry. This symmetry is subsequently broken by the mass terms.

The Table shows the subsets X_0 contributing to the fermion masses via $\langle A | X_0 | B \rangle$. The left hand column shows the dimensions of the $SU(4)_c \times SU(2)_L \times SU(2)_R$ representations to which the $\langle A | X_0 | B \rangle$ belongs. The right hand column gives the notation for the scalar vevs $\langle \phi_x \rangle$ labelled also by the SO(10) representation (10, 120 or 126) of which the X_0 is a member. For example the first pair of rows contain the neutral members of an $SU(2)_L \times SU(2)_R$ bi-doublet and so do the second, third and fourth pairs of rows. Further comments on this Table as well as details of how the terms in it give rise to the mass matrices are given in the next Section.

$SU(4)_c \times SU(2)_L \times SU(2)_R$	Γ products, X_0	scalar vev
$\{1, 2, 2\}$	χ_5	v_-^{10}
$\{1, 2, 2\}$	χ_5^\dagger	v_+^{10}
$\{1, 2, 2\}$	$\Gamma_7 \Gamma_8 \chi_5$	v_-^{120a}
$\{1, 2, 2\}$	$\Gamma_7 \Gamma_8 \chi_5^\dagger$	v_+^{120a}
$\{15, 2, 2\}$	$(\Gamma_1 \Gamma_2 + \Gamma_3 \Gamma_4 + \Gamma_5 \Gamma_6) \chi_5$	v_-^{120b}
$\{15, 2, 2\}$	$(\Gamma_1 \Gamma_2 + \Gamma_3 \Gamma_4 + \Gamma_5 \Gamma_6) \chi_5^\dagger$	v_+^{120b}
$\{15, 2, 2\}$	$(\Gamma_1 \Gamma_2 + \Gamma_3 \Gamma_4 + \Gamma_5 \Gamma_6) \Gamma_7 \Gamma_8 \chi_5$	v_-^{126a}
$\{15, 2, 2\}$	$(\Gamma_1 \Gamma_2 + \Gamma_3 \Gamma_4 + \Gamma_5 \Gamma_6) \Gamma_7 \Gamma_8 \chi_5^\dagger$	v_+^{126a}
$\{\overline{10}, 3, 1\}$	$\chi_1 \chi_2 \chi_3 \chi_4 \chi_5^\dagger$	v_-^{126b}
$\{10, 1, 3\}$	$\chi_1^\dagger \chi_2^\dagger \chi_3^\dagger \chi_4^\dagger \chi_5^\dagger$	v_+^{126b}

Table 1: SO(10) Clifford algebra operators giving rise to fermion masses

3 Yukawa coefficients, vevs, hermiticity

The mass terms shown in the Table break not only the SO(10) symmetry but also the Pati-Salam left-right symmetry since the operators in the middle column do not commute with all the Clifford algebra Pati-Salam generators. As a theory of particle masses, in the present context, they are only appropriate for use at high energy - say near the GUT energy - since the use of SO(10) Yukawas as in equation (5) is only appropriate near energies where SO(10) is a good symmetry. Such masses can ordinarily only be derived from experimental observation by the use of RGE equations such as those we shall make use of.

The symmetry breaking is fairly clear. For example the last row can not only supply the heavy Majorana neutrino mass necessary for the Type I see-saw but also breaks the $SU(2)_R$ symmetry because of the triplet component. It may be noted that the ninth row, necessary for the Type II seesaw breaks $SU(2)_L$ but because of the numerics of the neutrino masses only by a tiny amount. The combination $(\Gamma_1 \Gamma_2 + \Gamma_3 \Gamma_4 + \Gamma_5 \Gamma_6)$ occurring in rows 5,6,7,8 is proportional to the B-L generator and thus breaks the $SU(4)_c$ symmetry: $SU(4)_c \rightarrow SU(3)_c \times U(1)_{B-L}$. In addition the operators in rows 9 and 10 do not commute with $(B-L)$ so a non zero value of either of the corresponding vevs gives $SU(4)_c \rightarrow SU(3)_c$.

Thus if all the vevs in the Table are non-zero many symmetries are mul-

tiply broken.

The lines of the Table with one, three or five Γ_μ correspond respectively to subsets of the $\{10\}$, $\{120\}$ or $\{126\}$ algebras of $SO(10)$. Fixing conventions we illustrate by outlining the down quarks mass matrix calculation.

3.1 One Γ

. From the $\{10\}$ of $SO(10)$ these are the first two lines of the Table providing the colour singlet members of an $SU(2)_L \times SU(2)_R$ bi-doublet. To correspond to colour singlet, neutral vevs X_0 must be some combination of Γ_9 and Γ_{10} . The combinations χ_5 and χ_5^\dagger (being $(\Gamma_{10} \pm i\Gamma_9)/2$) are chosen with the corresponding vevs, $v_\pm^{10} \equiv \langle (\phi_{10} \pm i\phi_9) \rangle$.

Evaluation of the resulting expressions (Y_{AB}^{10} being Yukawa coefficients)

$$\langle \psi_{+A}^* | B_\Gamma C^{-1} (\chi_5 v_-^{10} + \chi_5^\dagger v_+^{10}) | \psi_{+B} \rangle Y_{AB}^{10} + h.c. \quad (6)$$

gives directly contributions to the mass matrix elements of the quarks, charged leptons and neutrinos as can be seen on inspection of eqns. (2),(3) and (4), using the operator algebra. Consider for example the contribution to the *two* down quark mass matrix elements A,B and B,A. For one Γ these are all included in

$$\begin{aligned} & [\langle \psi_{+A}^* | B_\Gamma C^{-1} \chi_5 | \psi_{+B} \rangle Y_{AB}^{10} + \\ & \langle \psi_{+B}^* | B_\Gamma C^{-1} \chi_5 | \psi_{+A} \rangle Y_{BA}^{10}] v_-^{10} + h.c. \end{aligned} \quad (7)$$

This is expressed in terms of left, d_L , and right, d_R , Weyl spinors using $C^{-1} = \sigma_2$ and equations (2),(3) with the correspondences $\tilde{\psi}_a = \sigma_2(d_{aR})^*$; $\psi_{a5} = d_{aL}$, a being the colour index. The result is

$$(d_{AR}^\dagger d_{BL} + d_{BR}^\dagger d_{AL}) v_-^{10} (Y_{AB}^{10} + Y_{BA}^{10}) \quad (8)$$

and the hermitian conjugate adds

$$(d_{BL}^\dagger d_{AR} + d_{AL}^\dagger d_{BR}) [v_-^{10} (Y_{AB}^{10} + Y_{BA}^{10})]^* \quad (9)$$

Obviously $Y_{AB} = Y_{BA}$ follows without loss of generality. These two equations can be combined and written in terms of 4-component Dirac spinors, d_A and d_B as

$$2[Re(v_-^{10} Y_{AB}^{10}) \bar{d}_A d_B + iIm(v_-^{10} Y_{AB}^{10}) \bar{d}_A \gamma_5 d_B], \quad (10)$$

$$2[Re(v_-^{10}Y_{AB}^{10})\bar{d}_B d_A + iIm(v_-^{10}Y_{AB}^{10})\bar{d}_B \gamma_5 d_A]. \quad (11)$$

and if $v_-^{10}Y_{AB}^{10}$ is real the pseudoscalar term vanishes and there is a real Dirac mass contribution to a symmetric flavour mass matrix. The other off-diagonal elements are of course completely similar and the diagonal contributions are real and scalar if $v_-^{10}Y_{AA}^{10}$ are real. Thus the flavour mass matrix is Hermitian. Also the mass contribution resulting conserves CP as well as P.

3.2 Three Γ

These give the $\{120\}$ anti-symmetric representations which are developed here as imaginary hermitian matrix representations.

(i) As shown in the Table there are two types of colour singlet neutral vevs associated with three Γ 's. In what follows the evaluation for the type

$$\Gamma_7 \Gamma_8 \chi_5, \Gamma_7 \Gamma_8 \chi_5^\dagger$$

is outlined. The flavour mass matrix elements for the down quarks are included in

$$\begin{aligned} & [\langle \psi_{+A}^* | B_\Gamma C^{-1} \Gamma_7 \Gamma_8 \chi_5 | \psi_{+B} \rangle Y_{AB}^{120} + \\ & \langle \psi_{+B}^* | B_\Gamma C^{-1} \Gamma_7 \Gamma_8 \chi_5 | \psi_{+A} \rangle Y_{BA}^{120}] v_-^{120a} + h.c. \end{aligned} \quad (12)$$

corresponding to equation (7). The total result of the calculation is

$$\begin{aligned} & -i[(d_{AR}^\dagger d_{BL} - d_{BR}^\dagger d_{AL})] v_-^{120a} (Y_{AB}^{120} - Y_{BA}^{120}) \\ & + i[(d_{BL}^\dagger d_{AR} - d_{AL}^\dagger d_{BR})] [v_-^{120a} (Y_{AB}^{120} - Y_{BA}^{120})]^* \end{aligned} \quad (13)$$

Obviously $Y_{AB}^{120} = -Y_{BA}^{120}$ follows without loss of generality. These two lines can be rearranged and written in terms of 4-component Dirac spinors, d_A and d_B as

$$-2i[Re(v_-^{120a} Y_{AB}^{120}) \bar{d}_A d_B + iIm(v_-^{120a} Y_{AB}^{120}) \bar{d}_A \gamma_5 d_B], \quad (14)$$

$$+ 2i[Re(v_-^{120a} Y_{AB}^{120}) \bar{d}_B d_A + iIm(v_-^{120a} Y_{AB}^{120}) \bar{d}_B \gamma_5 d_A]. \quad (15)$$

Suppose $v_-^{120a} Y_{AB}^{120}$ to be real. Then there is only a scalar mass term, the same applying to all the other off-diagonal contributions. The diagonal terms are anyway zero and thus the total contribution is imaginary anti-symmetric, so of hermitian flavour matrix form.

The extra factor i with a 3Γ operator such as $\Gamma_7 \Gamma_8 \chi_5$ is because $\Gamma_7 \Gamma_8 = i(\chi_4^\dagger \chi_4 - \chi_4 \chi_4^\dagger)$. Thus it might be thought that the association of hermiticity

with a purely scalar mass term is due to a particular choice of phase in $\Gamma_7\Gamma_8\chi_5$. This is not so; multiplication of that operator by $e^{i\phi}$ still yields the same association. This can be shown by explicit calculation but generally one can reason as follows.

The mass term in the Lagrangian of 3-flavoured chiral theories is

$$\psi_R^\dagger M \psi_L + \psi_L^\dagger M^\dagger \psi_R. \quad (16)$$

where M is a 3×3 flavour matrix and ψ_R, ψ_L are 2-component spinors with the three flavour index implicit. For Hermitian matrices, $M^\dagger = M$, this converts trivially into 4-component spinor, ψ , as a scalar, $\bar{\psi}M\psi$, and upon flavour diagonalization results in normal Dirac equations. However if M is not Hermitian

$$\psi_R^\dagger M \psi_L + \psi_L^\dagger M^\dagger \psi_R = \frac{1}{2}\bar{\psi}(M + M^\dagger)\psi - \frac{1}{2}\bar{\psi}\gamma_5(M - M^\dagger)\psi \quad (17)$$

and the pseudoscalar parity breaking γ_5 term intrudes upon the canonical Dirac equation. Conversely the presence of a γ_5 term breaks hermiticity.

(ii) Using the notation $\Gamma\Gamma \equiv \Gamma_1\Gamma_2 + \Gamma_3\Gamma_4 + \Gamma_5\Gamma_6$ it is seen from the Table that the other type of 3Γ operator is

$$\Gamma\Gamma\chi_5, \Gamma\Gamma\chi_5^\dagger.$$

For the down quarks this again yields the expressions (13)-(15) with v_-^{120b} instead of v_-^{120a} . Thus the association of hermiticity with a purely scalar mass term holds here too; and it equally applies for the masses of all particles.

For three Γ in both (i) and (ii) above and contrary to the cases of one Γ and five Γ the adopted purely scalar case violates CP because of the anti-symmetry of the $\{120\}$.

3.3 Five Γ

These arise from the $\{126\}$. As shown in the Table mass terms are generated by

$$\Gamma_7\Gamma_8\Gamma\Gamma\chi_5 \text{ and } \Gamma_7\Gamma_8\Gamma\Gamma\chi_5^\dagger$$

These indeed give results like the one Γ case in the respect that if $v_-^{126a}Y_{AB}^{126}$ is real the pseudoscalar term vanishes and there is a real scalar mass contribution to a symmetric flavour mass matrix. That is a scalar mass matrix contribution implies that it is hermitian, and vice versa. As before the same applies to all the particle masses given by these operators. As in the One Γ case the resulting contribution also conserves CP.

3.4 summary

Collecting now the contributions from just the first eight lines of the Table these also include the familiar operator coupling right chiral neutrinos to left chiral neutrinos which is here denoted by M^n . All these together result in the mass operators with obvious labels

$$M_{AB}^u = 2Y_{AB}^{10}v_+^{10} + 2Y_{AB}^{126}v_+^{126a} + 2iY_{AB}^{120}v_+^{120a} - 2iY_{AB}^{120}v_+^{120b} \quad (18)$$

$$M_{AB}^n = 2Y_{AB}^{10}v_+^{10} - 6Y_{AB}^{126}v_+^{126a} + 2iY_{AB}^{120}v_+^{120a} + 6iY_{AB}^{120}v_+^{120b} \quad (19)$$

$$M_{AB}^d = 2Y_{AB}^{10}v_-^{10} - 2Y_{AB}^{126}v_-^{126a} - 2iY_{AB}^{120}v_-^{120a} - 2iY_{AB}^{120}v_-^{120b} \quad (20)$$

$$M_{AB}^e = 2Y_{AB}^{10}v_+^{10} + 6Y_{AB}^{126}v_-^{126a} - 2iY_{AB}^{120}v_-^{120a} + 6iY_{AB}^{120}v_-^{120b} \quad (21)$$

where the Yukawas Y_{AB}^{10} and Y_{AB}^{126} are real and symmetric while the Y_{AB}^{120} are real and anti-symmetric. The vacuum expectation values v are also real to make all the above mass matrices hermitian.

It should be noted that the proof of hermiticity only evidently carries through in the fermion basis implicit in the present model; chiral rotations different for different flavours can change the coefficients of the scalar and pseudoscalar bilinears, changing the flavour mass matrices.

The last two 5Γ lines of the Table do not contribute mass to the quarks or charged leptons, but only to the neutrinos. This arises from evaluation of equations (3) using (1) and (2) and the associated particle assignments therein. These are given in the Appendix and associated with the left-right neutrinos of eqn(19). First order block diagonalization of the resulting 6×6 flavour mass matrix gives 3 light Majorana neutrinos and 3 heavy.

4 Mass Relations

The 16-plet of Weyl fermions eqns.(1,2) has chiral neutrinos, n_L , $(n_R)^c$. We denote the 4-component neutrinos corresponding to n_L , n_R as ν_L , N_R respectively (see the Appendix for further details). The see-saw hypothesis assigns a large mass to N_R through a Majorana mass term, with flavour matrix (arising from eqn.(3) and the last line of the table) here denoted M . These left and right neutrinos couple together with flavour matrix M^n , given in eq.(19), analogous to the quark and charged lepton mass matrices. The penultimate line of the Table gives rise to a Majorana mass term for ν_L . The

flavour matrix for this, numerically very small compared to M , is denoted by m . Diagonalization of the resulting matrix

$$M^{neutrinos} = \begin{bmatrix} m & M^n \\ M^{nT} & M \end{bmatrix} \quad (22)$$

results in 3 light physical Majorana neutrinos and 3 heavy physical neutrinos in the top left and bottom right respectively of $M^{neutrinos}$. The light neutrino Lagrangian has: (i) a contribution having P and CP invariance containing a 3×3 Hermitian flavour mass matrix; (ii) a contribution violating both P and CP containing a 3×3 imaginary non-Hermitian flavour mass matrix. This latter term arises from the original ('Type I') seesaw mechanism and is the only P violating term at high energy in the resulting mass Lagrangian of the model.

For analyzing, as now follows, the mass relations of the quarks and charged leptons the notation is simplified, dropping the flavour indices from the Yukawa matrices, Y , and defining new matrices which incorporate the scalar vacuum expectation values (such as v_{\pm}^{10} for those in the $\{10\}$).

$$h = Y^{10}v_{-}^{10}, f = Y^{126a}v_{-}^{126}, g = Y^{120}v_{-}^{120a}. \quad (23)$$

Certain ratios of the real vacuum expectation values, required for the mass equations (26) to (29) below, are

$$\begin{aligned} r_h &= v_{+}^{10}/v_{-}^{10}, r_f = v_{+}^{126a}/v_{-}^{126a}, r_g = v_{+}^{120a}/v_{-}^{120a}, \\ r_1 &= v_{-}^{120b}/v_{-}^{120a}, r_2 = v_{+}^{120b}/v_{+}^{120a}. \end{aligned} \quad (24)$$

In addition for the Majorana neutrino matrices $M = r_M f$ and $m = r_m f$

$$r_M = v_{+}^{126b}/v_{-}^{126a}, r_m = v_{-}^{126b}/v_{-}^{126a} \quad (25)$$

With these notations the quark and lepton mass matrix equations are

$$M^d = h - f - ig(1 + r_1) \quad (26)$$

$$M^e = h + 3f - ig(1 - 3r_1) \quad (27)$$

$$M^u = r_h h + r_f f + ir_g g(1 - r_2) \quad (28)$$

$$M^n = r_h h - 3r_f f + ir_g g(1 + 3r_2) \quad (29)$$

where the mass matrices are hermitian with h, f being real symmetric and g real antisymmetric. As noted in the introduction the hermiticity is a significant

difference from the majority of previous papers. (Allowing for changes due to conventions the part of these equations in h and f are recognisably the same as those written in very many previous papers such as references [2] - [8]. On the other hand the terms in g are formally, and physically, different as they involve the ratios r_1 and r_2 arising from the equations of section III.D.)

Putting aside neutrino masses and mixing to be considered later, the present data, some of it being significantly only approximate, is 9 quark and charged lepton masses, 3 CKM matrix angles and 1 phase.

To make use of the CKM matrix ($V = U_d^\dagger U_u$) data it is a common device, when possible, to take a basis in which either the d-quark matrix is real diagonal (implying U_d is unity) or the u-quark matrix is (so U_u is unity). Then either U_u or U_d^\dagger respectively is the CKM matrix. While this might be done in the general case of the present model the unitary matrix required to change the basis generally bestows imaginary parts on the real matrices h, f, g , thus upsetting a simplifying feature of the model. However we can avoid this upset and shall make use of this device by considering some special cases.

4.1 Special Cases

To retain those features and the (relative) simplicity of numerical calculations there are two special cases of the vevs associated with the two couplings (which we have denoted as 120a and 120b respectively) in the $\{120\}$. These are (i) $v_+^{120b} = v_+^{120a} \Rightarrow r_2 = 1$, making M^u real symmetric, diagonalisable by a real orthogonal change of basis; and (ii) $v_-^{120b} = -v_-^{120a} \Rightarrow r_1 = -1$, making M^d real symmetric, diagonalisable by a real orthogonal change of basis. The latter special case involves importing a relative phase of π which however preserves the reality conditions of the model. Taking either of these special cases, with their associated change of basis, preserves the real symmetric (anti-symmetric) nature of the matrices $h, f, (g)$.

To illustrate the numerical evaluation consider the special case (ii) which will be used in the following section on neutrino masses and mixing. The mass matrix equations simplify to

$$M^d = h - f \tag{30}$$

$$M^e = h + 3f - 4ig \tag{31}$$

$$M^u = r_h h + r_f f + ir_g g(1 - r_2) \tag{32}$$

$$M^n = r_h h - 3r_f f + ir_g g(1 + 3r_2) \tag{33}$$

The first three equations yield the mass relation

$$M^e = xM^d + yRe(M^u) + izIm(M^u) \quad (34)$$

which, equating matrix coefficients in the real and imaginary parts of equations (30), (31), (32), has

$$\begin{aligned} x &= (r_f - 3r_h)/(r_h + r_f), \\ y &= 4/(r_h + r_f), \\ z &= -4/r_g(1 - r_2). \end{aligned} \quad (35)$$

The three matrices on the right hand side of equation (34) are evaluated as follows in terms of the quark masses called here d, s, b and u, c, t , and the CKM matrix V . Now $M^d, Re(M^u), Im(M^u)$ are written respectively as

$$\begin{bmatrix} d & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & b \end{bmatrix}, \begin{bmatrix} w_1 & u_1 & u_3 \\ u_1 & w_2 & u_2 \\ u_3 & u_2 & w_3 \end{bmatrix}, \begin{bmatrix} 0 & v_1 & v_3 \\ -v_1 & 0 & v_2 \\ -v_3 & -v_2 & 0 \end{bmatrix}.$$

As stated above since the basis is diagonal in the down quarks and all the matrices are Hermitian then M^u is diagonalized by the CKM matrix, V , giving the real parameters u_i, v_i, w_i in terms of u, c, t and the CKM matrix by

$$M^u = V^\dagger M_{diag}^u V \quad (36)$$

the diagonal matrix M_{diag}^u having elements u, c, t .

The choice of the CKM matrix is the additional input at this stage.

Given a matrix, (i) its trace, (ii) the trace of the inverse and (iii) the determinant are each invariant under a unitary transformation. This leads to the known way of solving equations such as (34) since a unitary transformation can transform the Hermitian M^e into a diagonal matrix of the charged lepton masses. Then we equate the three invariants of the two sides of equation (34). For the right hand side (i) gives an expression linear in x, y while (ii) being equivalent to the sum of the 2×2 diagonal sub-matrices gives a quadratic in (x, y, z) but with z only occurring as z^2 and (iii) yields a cubic but again with z only occurring as z^2 .

The coefficients are real and elimination gives a cubic equation, either in x or y , which can be solved by an analytic expression and on evaluation yields numbers for the set x, y, z^2 . Only those sets with x, y real and $z^2 > 0$ are acceptable solutions of equation (34).

5 Neutrino Masses and Mixings

Continuing with the special case where $r_1 = -1$ the light neutrino masses and mixing are due for consideration. The approach is first to identify the three remaining free real parameters of the model. Then to choose a possible set of 3 neutrino masses which are in accord with mass data and search for a solution of those parameters (in principle there could be more than one) for which the model gives the chosen masses. This can be repeated for various choices of light neutrino masses. For each solution set the model is fixed with all parameters determined by the assumed quark, charged lepton and light neutrino masses and CKM matrix. Each parameter-fixed model predicts an MNS mixing matrix which can be considered for plausibility.

So far use has been made of 13 assumed data points - 9 quark and charged lepton masses, 3 CKM angles and 1 phase - albeit some of these are subject to considerable error. Using equations (30),(31),(32),(36) and a basis diagonal in the mass matrix M^d the hermitian quark and charged lepton mass matrices M^d, M^u, M^e have been synthesized from the 13 data points. An alternative description, in terms of the quantities on the RHS of equations (30),(31),(32),(33) is that the real matrices h, f, g along with the real parameters $r_h, r_f, r_g(1 - r_2)$ have been constructed. (There seem to be 15 constructed parameters from 13 data points. However these 15 are not independent because eq.(36), arising from the special assumptions, produces 9 parameters from 7 data.)

Turning now to the neutrinos, a little manipulation of the equations yields the neutrino mass matrix of eq.(33) as

$$M^n = \frac{1-x}{y}M^d - \frac{2+x}{y}4\tilde{M} + i\frac{1+3r_2}{1-r_2}Im(M^u) \quad (37)$$

$$\tilde{M} \equiv (Re(M^e) - M^d)/4 = f = M/r_M = m/r_m. \quad (38)$$

In eq.(37) r_2 occurs. It is a free parameter since only $r_g(1 - r_2)$ has been constructed.

The 'seesaw' mass matrices m and M of eq. (22) (arising from the {126} representations of the last two rows of the Table) are proportional to the matrix f as displayed in (38). Thus there are 2 more free parameters, r_m and r_M , in addition to r_2 , leaving the model with just 3 so far undetermined real (and dimensionless) parameters. These can in principle be fixed by fitting 3 given light neutrino masses. Since data is only known on mass differences exploration requires postulation of one light neutrino mass.

By a redefinition of the neutrino states, working to first order in the numerically small matrix $\eta = M^n M^{-1}$, as shown in the Appendix, $M^{neutrinos}$ can be transformed to block diagonal form

$$\begin{bmatrix} m_\nu & 0 \\ 0 & M \end{bmatrix}.$$

The 3×3 matrix of the small mass neutrinos is given by the usual seesaw formula (Type II and Type I) $m_\nu = m - M^n M^{-1} M^{nT}$. For the present special case the matrix $m = r_m f$, contributed wholly by the $\{126\}$, is hermitian, being real and symmetric. The Type I contribution, $-M^n M^{-1} M^{nT}$, though composed from hermitian matrices is neither real nor hermitian. However it is symmetric thus m_ν can be written in terms of real symmetric matrices m_1, m_2 as $m_\nu = m_1 + im_2$. The eigenvalues of the hermitian matrix

$$m_\nu m_\nu^\dagger = (m_1 + im_2)(m_1 - im_2) \quad (39)$$

are the neutrino masses squared.

The questions now are: (i) are the 3 free parameters sufficient to fit some possible sets of experimental neutrino masses; (ii) if so is agreement also found with our knowledge of the MNS matrix

5.1 An Example

The Yukawa matrices, Y^{10}, Y^{126}, Y^{120} have been assumed throughout to be appropriated to the 10, 126, 120 SO(10) representations respectively. While this paper is not committed to a GUT model nevertheless the quark and lepton masses used should be those appropriate to a high energy where parity and SO(10) are restored, thus involving extrapolation by renormalization group equations. There are well known (and well used) extrapolations by Das and Parida [14] including renormalization group equations of the standard (nonSUSY) model, the 2 Higgs doublet model and the minimum supersymmetric model. These have the general feature of such extrapolations of shifting the quark masses by considerably more than the relative shift of the lepton masses. For the light neutrino masses the present low energy data is here used as a guide.

Possible sets of extrapolated quark and charged lepton masses and CKM matrix using the results of Das and Parida [14] have been constructed and used in various papers. We make use of some of those previous works by

adopting the data sets (appropriate to an energy of 2×10^{16} GeV of (i) Goh et al. [2] and (ii) Bertolini et al.[7]). However it is not at all the purpose in this paper to make an assiduous search for solutions plausible on some criteria. Rather it is to sample sparsely to illustrate some possibilities albeit in a special case ($r_1 = -1$) of the original model.

For the present experimental data the 3-neutrino mixing scheme reviewed by B. Kayser [13] is used. The neutrinos being named as 1,2,3 the following central values of the difference of squared masses are adopted

$$(\Delta m^2)_{21} = 8.0 \times 10^{-5} eV^2, (\Delta m^2)_{32} = 2.5 \times 10^{-3} eV^2,$$

along with the assumption of a neutrino hierarchy $m_{\nu 1}^2 < m_{\nu 2}^2 < m_{\nu 3}^2$ so that

$$m_{\nu 2}^2 = m_{\nu 1}^2 + (\Delta m^2)_{21} \quad (40)$$

$$m_{\nu 3}^2 = m_{\nu 2}^2 + (\Delta m^2)_{32}. \quad (41)$$

The computing code calculates theoretical values of these 3 neutrino masses squared: $\mu_i^2(r_2, r_m, r_M)$, each of these three being functions of the free parameters r_2, r_m, r_M . It is necessary to find values of (r_2, r_m, r_M) so that the μ_i^2 are equal to the $m_{\nu i}^2$, ($i = 1, 2, 3$). This is done in an obvious way by inventing and optimizing a function whose extreme value (say zero) is reached when $\mu_i^2 = m_{\nu i}^2$ for all ($i = 1, 2, 3$), thus achieving an exact solution. The principle of this procedure is not the same as finding the 'best fit' (for example by χ^2 minimization) of many parameters. Because of the complication of the numerical calculation of the neutrino masses it is almost necessary to use a non-derivative method. So a non-derivative simplex method [15], operating in the three-dimensional space of (r_2, r_m, r_M) is used to find any zeros of the chosen function. The computer search is specialised to those regions of small r_m and large r_M suitable to produce small neutrino masses.

(i) The paper of Goh et al.[2] uses the Das and Parida extrapolation of masses to 2×10^{16} GeV by the MSSM RGE ($\tan(\beta) = 10$ in Table II of [14]) and also quotes the real part of the CKM m matrix. These are used as data input in the present example. Using the Wolfenstein parametrization gives Im(CKM) as

$$\begin{bmatrix} 0 & 0 & -.00320 \\ 0 & -.00059 & .00074 \\ -.00328 & 0 & 0 \end{bmatrix}, \quad (42)$$

In all this $m_{\nu 1}^2$ is a quantity of choice being part of any postulate on neutrino masses. A first idea is that it should be small but not negligible,

say of the order of $1.0 \times 10^{-5} eV^2$. In this region the optimization code gives the desired equality to very great accuracy. For $m_{\nu 1}^2 = 2.0 \times 10^{-5} eV^2$, $m_{\nu 2}^2 = 1.0 \times 10^{-4} eV^2$, $m_{\nu 3}^2 = 2.6 \times 10^{-3} eV^2$ then the dimensionless free parameters of equations (37) and (38) have the values

$$r_2 = 3.13, r_m = 2.1 \times 10^{-9}, r_M = 5.9 \times 10^{15}. \quad (43)$$

Having fixed the free parameters, the only extra input having been the partly hypothesized neutrino masses, it is then possible to calculate the MSN matrix. Rather surprisingly one finds good agreement with presently accepted features of this matrix.

The charged lepton mass matrix, M^e , (now calculable) is hermitian in the model and so diagonalizable by a unitary transformation

$$X_e^\dagger M^e X_e = M_{diag}^e \quad (44)$$

The light neutrino mass matrix, $m_\nu = m_1 + im_2$, and the matrix for the masses squared $(m_1 + im_2)(m_1 - im_2)$ is hermitian and can be diagonalized by a unitary transformation as

$$X_\nu^\dagger (m_1 + im_2)(m_1 - im_2) X_\nu. \quad (45)$$

The diagonalization of m_ν requires a bi-unitary transformation as

$$X_\nu^\dagger (m_1 + im_2) Y_\nu. \quad (46)$$

The MNS matrix, similarly to the CKM matrix, is defined as

$$U_{MNS} = X_e^\dagger X_\nu. \quad (47)$$

and U can be calculated in the present model using the values of the 3 free parameters fixed by using values of the neutrino masses as discussed above. In this particular case the result for the matrix of moduli squared of the elements of U ($|U_{e\nu}|^2$) is

$$\begin{bmatrix} .638 & .344 & .017 \\ .260 & .331 & .409 \\ .102 & .325 & .573 \end{bmatrix}, \quad (48)$$

bearing a distinct resemblance to the postulated 'ideal' structure of this matrix in tri-bimaximal mixing [16]:

$$\begin{bmatrix} 2/3 & 1/3 & 0 \\ 1/6 & 1/3 & 1/2 \\ 1/6 & 1/3 & 1/2 \end{bmatrix}, \quad (49)$$

It may be noted that the 13, 23 and 12 elements are clearly within the range of many analyses of the experiments [16]. (It is interesting that in this solution the mass matrix elements contribution from Type II are much greater than those from Type I except for the (3,3) where the contributions are of the same order of magnitude.) Similar results hold for all values of $m_{\nu 1}^2$ between about 1.6×10^{-5} and 2.4×10^{-5} in eV^2 .

(ii) Bertolini et al. [7], making some use of the Das and Parida [14] extrapolation of masses to 2×10^{16} GeV by the MSSM RGE ($\tan(\beta) = 10$) have given quark and charged lepton masses and 3 CKM angles plus the phase angle. Bertolini et al. [7] have revised the central values of the extrapolated masses of the lightest quarks to 0.55, 1.24, 21.7 MeV whereas Goh et al. [2], as used in (i), have 0.72, 1.5, 30.0 MeV. The extrapolated masses of the three heaviest quarks and the charged leptons remain the same as those in [2]. Some off-diagonal elements of the CKM matrix also display some non-trivial differences from those of Goh et al [2] used in (i).

In the fitting, of the theory to the pseudo-physical neutrino masses, some samples of $m_{\nu 1}^2$ were taken in and around the same region as in (i) above. Any that succeeded in fitting yielded significantly different parameters from those in (i) resulting in spectacularly implausible MSN matrices. Further numerical investigation seemed to show that each one of the data changes mentioned above had influence on the results. That is results are sensitive to changes in the least well known data. It should be emphasized that neither in case (i) nor case (ii) was there attempted extensive investigation of very many neutrino spectra.

6 Summary

The model makes use of SO(10) but with multiplets restricted in number and kind.

Firstly the philosophy is to adopt the 16-plet of fermions as the particles we know and only consider couplings arising from $16 \times 16 = 10 + 120 + \overline{126}$. Secondly to restrict those couplings so that the resulting mass matrices are hermitian at high energy; an argument is given that within SO(10) this can be formulated naturally; also the restoration at high energy of parity invariance of Lagrangian mass terms implies hermiticity. The scalar vacuum expectation values are classified in the Pati-Salam subgroup of SO(10) using the Clifford algebra representation; this distinguishes two realisations of the $\{120\}$ asso-

ciated with different vacuum expectation values. Assuming the values for the quark and charged lepton masses and the CKM angles and phase leaves just 4 undetermined real dimensionless parameters in the theory. Fixing one of these parameters to a special value, thus simplifying the calculation and the range of results, leaves just 3 free parameters. To fix these the hierarchical hypothesis for light neutrino masses is adopted, together with specifying the lowest neutrino mass (at various values) to give the 3 light neutrino masses in accord with existing data on neutrino mass squared differences. The theory then predicts the MSN matrix. In the case of one set of quark and charged lepton masses and CKM matrix [2], and a range of lowest neutrino masses, the MSN matrix is, rather surprisingly, in accord with the tri-bimaximal mixing suggested by the data [16]. This result does not hold for another data set [7], for the same physical quantities, that was tried. Solutions thus appear sensitive to changes in the less well known physical quantities.

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APPENDIX

In addition to the neutrino content of the standard model which has just left chiral (and massless) neutrinos, the 16-plet of $SO(10)$ has both left chiral, n_L , and right chiral, n_R , neutrinos. These occur in the 16-plet vector (2) as $\tilde{\psi}_5$ and $-\psi_o^c$ respectively, being 2-component Weyl spinors..

The matrix elements (4) give rise to three mass matrices for the neutrinos. Two of these are the self couplings of n_L and n_R respectively. Both of these are Majorana mass terms. The third couples n_L and n_R and gives rise to the mass term M^n which appears in equations (29) and (33). The terms are as follows.

(i) The self couplings of n_R arise from $\chi_1^\dagger \chi_2^\dagger \chi_3^\dagger \chi_4^\dagger \chi_5^\dagger$ as in the last line of the table. They are

$$(n_{AR})^{cT} C^{-1} (n_{BR})^{cT} v_+^{126b} Y_{AB}^{126} + hc$$

where A, B are generation indices. These can be put into 4 component spinor notation by defining 4 component right chiral spinors

$$N_{AR}^T \equiv (0, n_{AR})$$

giving the (Majorana) mass terms as

$$\begin{aligned} & \frac{1}{2} \overline{N_{AR}^c} N_{BR} M_{AB} + hc \\ M_{AB} &= 2v_+^{126b} Y_{AB}^{126} \end{aligned} \tag{50}$$

Since v_+^{126b} and Y_{AB}^{126} are real M is a real symmetric matrix.

(ii) The self couplings of n_L arise from $X = \chi_1 \chi_2 \chi_3 \chi_4 \chi_5^\dagger$ as in the penultimate line of the Table. Analogously to (i) above the resulting 2-component (Weyl) spinor results can be expressed in 4 component spinor terms by defining

$$\nu_{AL}^T \equiv (n_{AL}, 0)$$

giving the (Majorana) mass as

$$\begin{aligned} & \frac{1}{2} \overline{\nu_{AL}^c} \nu_{BL} m_{AB} + hc \\ m_{AB} &= 2v_-^{126b} Y_{AB}^{126} \end{aligned} \tag{51}$$

Since v_-^{126b} and Y_{AB}^{126} are real m is a real symmetric matrix.

(iii) The couplings of n_R to n_L arise from X as in the first 8 lines of the Table. In 4-component spinors these give mass terms

$$\overline{\nu_{AL}} N_{BR} M_{AB}^n + hc$$

where M^n is the hermitian mass matrix of equation (29).

These neutrino mass terms can be written in matrix form, using $\overline{\nu_L} M^n N_R = \overline{N_R^c} M^{nT} \nu_L^c$ [17], as

$$\frac{1}{2} \begin{bmatrix} \overline{\nu_L} & \overline{N_R^c} \end{bmatrix} \begin{bmatrix} m & M^n \\ M^{nT} & M \end{bmatrix} \begin{bmatrix} \nu_L^c \\ N_R \end{bmatrix} + hc.$$

The matrix M is very large compared to M^n and m . So, working to first order in M^{-1} , the seesaw method can be implemented by defining

$$L = \nu_L + (M^{-1}m)_R^c N, R^c = N_R + (M^{-1T}m)\nu_L \quad (52)$$

noting that M and m are hermitian matrices. The neutrino mass terms become

$$\frac{1}{2} \begin{bmatrix} \overline{L} & \overline{R^c} \end{bmatrix} \begin{bmatrix} m_\nu & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} L^c \\ R \end{bmatrix} + hc.$$

$$m_\nu = m + m_\nu^I \quad (53)$$

$$m_\nu^I = -M^n M^{-1} M^{nT} \quad (54)$$

Eqn(54) gives the original seesaw term, often known as Type I seesaw. m is hermitian and m_ν^I is complex symmetric. Denoted (m_ν, M) are the flavour mass matrices of the (light, heavy) Majorana neutrinos with Majorana fields [17]

$$\nu = L + L^c, N = R^c + R. \quad (55)$$

with the mass terms for the light neutrinos being

$$\begin{aligned} & \frac{1}{2} \overline{\nu} m_\nu (1 + \gamma_5) \nu + hc \\ & = \overline{\nu} m \nu + \overline{\nu} [Re(m_\nu) + i\gamma_5 Im(m_\nu)] \nu. \end{aligned} \quad (56)$$

In eqn(56) the two first terms, containing hermitian matrices, are P and CP invariant but the $i\gamma_5$ symmetric matrix term violates CP as well as P. Thus P and CP violation in the light Majorana neutrino mass terms arise solely from the original seesaw mechanism involving the non-hermitian product of 3 hermitian matrices.

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